SYDNEY GRAMMAR SCHOOL

2013 Half-Yearly Examination

FORM VI
MATHEMATICS 2 UNIT

Thursday 21st February 2013

General Instructions

- Writing time — 2 hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.
- A list of standard integrals is provided at the end of the examination paper.

Total — 85 Marks

- All questions may be attempted.

Section I – 10 Marks

- Questions 1–10 are of equal value.
- Record your solutions to the multiple choice on the sheet provided.

Section II – 75 Marks

- Questions 11–15 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your candidate number on each booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Place your multiple choice answer sheet inside the answer booklet for Question Eleven.
- Write your candidate number on this question paper and submit it with your answers.

Checklist

- SGS booklets — 5 per boy
- Multiple choice answer sheet
- Candidature — 96 boys

Examiner
SG

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

Which of the following is a primitive of $\sqrt{x}$?

(A) $\dfrac{3}{2}x^{\frac{3}{2}}$

(B) $\dfrac{2}{3}x^{\frac{3}{2}}$

(C) $\dfrac{1}{2}x^{-\frac{1}{2}}$

(D) $-\dfrac{1}{2}x^{-\frac{1}{2}}$

QUESTION TWO

What is the value of the definite integral $\displaystyle\int_{-1}^{2} x^2\,dx$?

(A) 3

(B) -3

(C) $\dfrac{7}{3}$

(D) $-\dfrac{7}{3}$

Exam continues next page ...

QUESTION THREE

A point $P(x, y)$ moves so that it is always equidistant from the points $A(0, 0)$ and $B(5, 5)$. Which of the following best describes the locus of P?

(A) A line

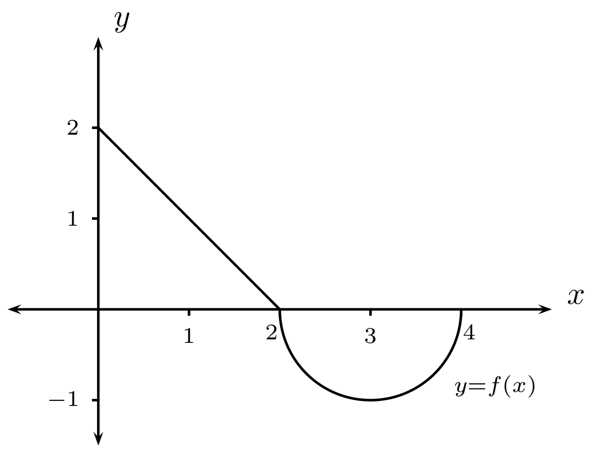
(B) A circle

(C) A parabola

(D) A hyperbola

QUESTION FOUR

The graph of $y = f(x)$ is shown below. It consists of a straight line section and a semicircle. What is the value of the definite integral $\displaystyle\int_0^4 f(x)\,dx$?



(A) $2 + \pi$

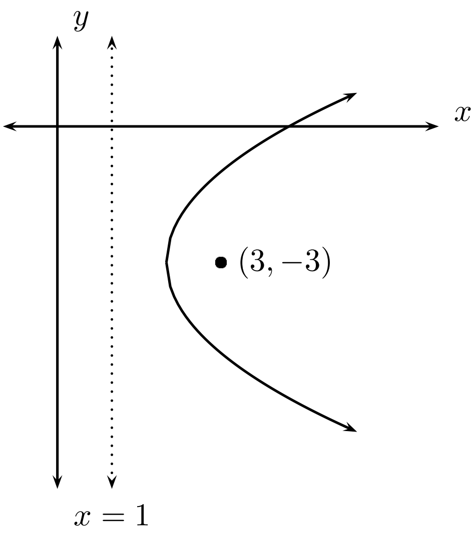
(B) $2 - \pi$

(C) $2 + \dfrac{\pi}{2}$

(D) $2 - \dfrac{\pi}{2}$

QUESTION FIVE

A parabola has its focus at $(3, -3)$ and directrix at $x = 1$. What is the equation of this parabola?



(A) $(y + 3) = 4(x - 2)^2$

(B) $(y + 3)^2 = 4(x - 2)$

(C) $(y - 3)^2 = 4(x + 2)$

(D) $(y - 3) = 4(x + 2)^2$

QUESTION SIX

The graph of $y = e^x$ is translated two units to the right. Which of the following represents the new function?

(A) $y = e^{x+2}$

(B) $y = e^{x-2}$

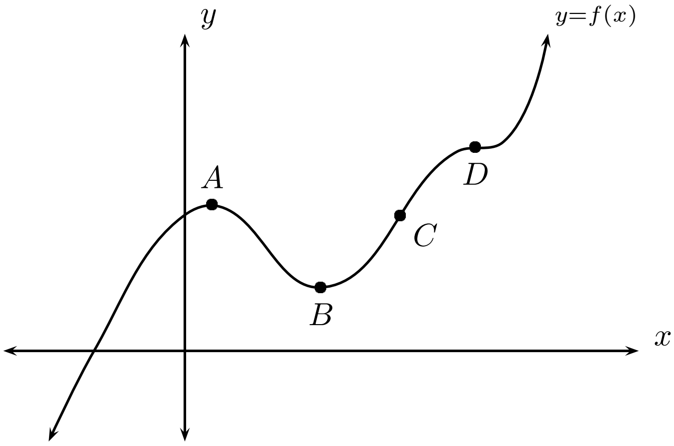
(C) $y = e^x + 2$

(D) $y = e^x - 2$

Exam continues next page ...

QUESTION SEVEN

For $y = f(x)$ graphed below, which of the labelled points satisfies $\dfrac{dy}{dx} = 0$ and $\dfrac{d^2y}{dx^2} < 0$?



(A) A

(B) B

(C) C

(D) D

QUESTION EIGHT

Which of the following is not true of the function $f(x) = e^x + 3$?

(A) The first derivative is e^x.

(B) The function is always increasing.

(C) The function has its y-intercept at $(0, 1)$.

(D) A primitive of the function is $e^x + 3x$.

QUESTION NINE

The function $f(x) = (x-2)^2(x+7)$ has the following first and second derivatives:

$$f'(x) = 3(x-2)(x+4)$$
$$f''(x) = 6(x+1)$$

Which of the following statements is false?

(A) $f(x)$ has stationary points at $x = 2$ and $x = -4$.

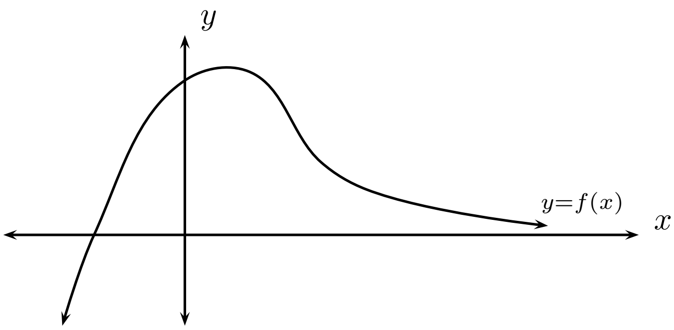
(B) $f(x)$ is concave up for $x < -1$.

(C) $f(x)$ has x-intercepts at $x = 2$ and $x = -7$.

(D) $f(x)$ is a cubic function.

QUESTION TEN

A function $y = f(x)$ is graphed below. The x-axis is an asymptote for the function.



Which of the following statements is true?

(A) $f(x)$ has a global minimum at $y = 0$.

(B) $f(x)$ has an asymptote at $x = 0$.

(C) $f(x)$ has a single point of inflexion.

(D) $f(x)$ has two stationary points.

End of Section I

Exam continues next page ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION ELEVEN (15 marks) Use a separate writing booklet. **Marks**

(a) Calculate $\dfrac{3e^2}{5}$, correct to three decimal places. **1**

(b) Simplify $\dfrac{(e^x)^3}{e^{2x}}$. **2**

(c) (i) Sketch the locus of a point P which moves so that it is always at a fixed distance of two units below the x-axis. **1**

(ii) Write down the equation of the locus. **1**

(d) A parabola has equation $x^2 = -4y$.

(i) Write down the coordinates of the vertex. **1**

(ii) Write down the coordinates of the focus. **1**

(iii) Write down the equation of the directrix. **1**

(iv) Sketch the parabola, showing all the features found in (i) to (iii). **1**

(e) Differentiate the following with respect to x:

(i) $5x^3 - 3x^2 + 9$ **1**

(ii) $4e^{5x}$ **1**

(iii) $(2 - 3x)^4$ **2**

(f) Find a primitive for each of the following:

(i) $x^2 - 2$ **1**

(ii) $x^{-\frac{1}{2}}$ **1**

Exam continues overleaf . . .

QUESTION TWELVE (15 marks) Use a separate writing booklet. **Marks**

(a) Expand and simplify $(e^x + e^{-x})^2$. **2**

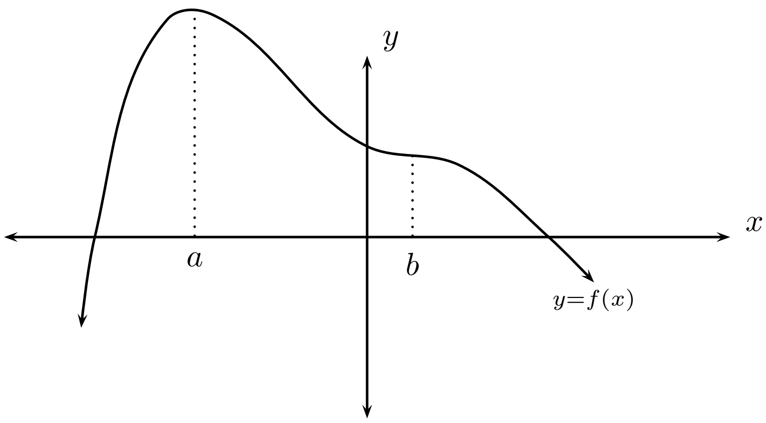
(b) Evaluate the following definite integrals:

(i) $\displaystyle\int_{-2}^{2} (3x^2 + 2x)\, dx$ **2**

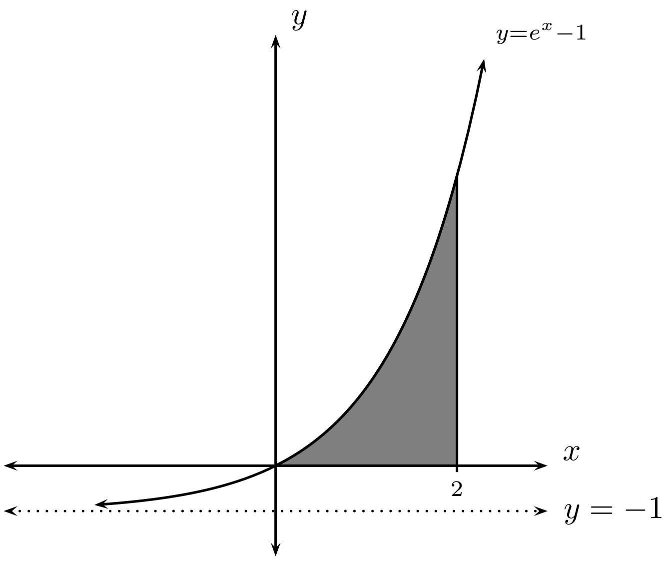
(ii) $\displaystyle\int_{1}^{3} \frac{2}{x^2}\, dx$ **2**

(c) Given $f'(x) = e^x + 2x$, find $f(x)$ if $f(0) = 0$. **2**

(d) The following diagram shows the graph of $y = f(x)$. A maximum occurs when $x = a$ and a stationary point of inflexion when $x = b$. Sketch a possible graph of $f'(x)$. **2**



(e) Find the area of the region bounded by the curve $y = e^x - 1$, the x-axis and the line $x = 2$, as shown in the diagram below. Express your answer in terms of e. **2**



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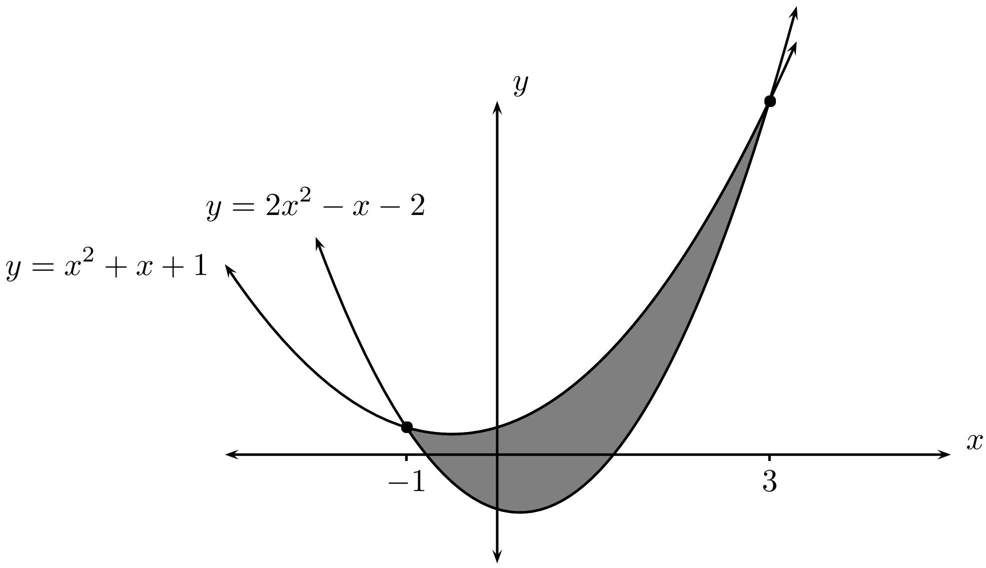
QUESTION TWELVE (Continued)

(f) The curves $y = x^2 + x + 1$ and $y = 2x^2 - x - 2$ meet at two points whose x-coordinates are $x = -1$ and $x = 3$. **3**



Find the area of the shaded region enclosed between the two curves, as shown in the diagram above.

QUESTION THIRTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) Consider the curve $y = 3x^3 - 9x + 3$.

(i) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. **2**

(ii) Find all the stationary points of the curve. **2**

(iii) Determine the nature of each stationary point. **2**

(iv) Sketch the curve, clearly indicating all the stationary points and the y-intercept. **2**

Note, you are not required to find the x-intercepts or any points of inflexion.

(b) Evaluate $\displaystyle\int_{-2}^{2} e^{2x+1}\,dx$. **2**

(c) Using Simpson's rule with five function values, estimate $\displaystyle\int_{1}^{5} \frac{1}{x}\,dx$ correct to three decimal places. **2**

(d) Find the equation of the tangent to the curve $y = e^{3x}$ at the point where $x = 0$. **3**

Exam continues overleaf ...

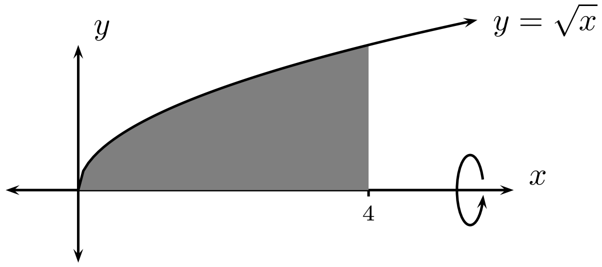
QUESTION FOURTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) The cost, C dollars, of running a vehicle at an average speed of v km/h is given by **3**

$$C = \frac{2}{5}v + 2000v^{-1}, \quad \text{where } v > 0.$$

For what average speed will the cost be minimised?

(b) The region bounded by the curve $y = \sqrt{x}$, the x-axis and the line $x = 4$, is shown below. **2**



Find the volume of the solid generated when this region is rotated about the x-axis.

(c) A function $f(x)$ has second derivative $f''(x) = 20(x-1)^2(x-4)$. Show that $f(x)$ has only one point of inflexion. **2**

(d) Differentiate the following, leaving your answers in simplest form.

(i) $y = (2x-1)e^x$ **2**

(ii) $y = \dfrac{e^x}{2x+3}$ **3**

(e) (i) Differentiate $y = e^{x^3}$. **1**

(ii) Hence evaluate the definite integral $\displaystyle\int_0^1 3x^2 e^{x^3}\,dx$. **2**

QUESTION FIFTEEN (15 marks) Use a separate writing booklet. **Marks**

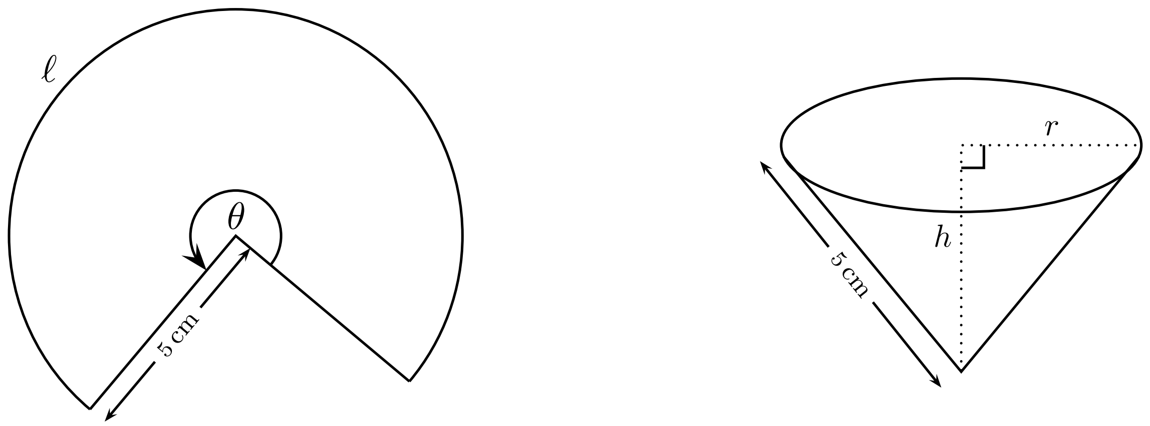
(a) Consider the function $f(x) = \dfrac{1}{(x+4)^2}$.

(i) Find $f''(x)$. **2**

(ii) Explain why $y = f(x)$ is concave up for all real x except $x = -4$. **1**

(b) Show that the function $y = xe^{-2x}$ satisfies the equation $y'' + 4y' + 4y = 0$. **3**

(c) A sector with radius 5 cm, arc length ℓ cm and angle θ degrees at its centre is bent to form a cone, as shown in the diagram below. The resultant cone has base radius r cm and height h cm.



(i) Show that $\ell = \dfrac{\pi\theta}{36}$. **1**

(ii) Hence show that $r = \dfrac{\theta}{72}$. **1**

(iii) Show that $h = \sqrt{25 - \left(\dfrac{\theta}{72}\right)^2}$. **1**

(iv) Construct an equation for the volume of the cone $V\text{ cm}^3$ as a function of θ only. **2**

(v) Find, to the nearest degree, the value of θ for which the volume of the cone is maximised. **4**

End of Section II

END OF EXAMINATION

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The following list of standard integrals may be used:

$$\int x^n\,dx = \frac{1}{n+1}\,x^{n+1}, \;\; n \neq -1; \;\; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \;\; x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}\,e^{ax}, \;\; a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \;\; a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \;\; a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \;\; a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \;\; a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}\,, \;\; a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}\,, \;\; a > 0, \;\; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \;\; x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

$$\text{NOTE}: \;\; \ln x = \log_e x, \;\; x > 0$$

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FORM VI
MATHEMATICS 2 UNIT
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- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

CANDIDATE NUMBER: ....

Question One

A ◯ B ◯ C ◯ D ◯

Question Two

A ◯ B ◯ C ◯ D ◯

Question Three

A ◯ B ◯ C ◯ D ◯

Question Four

A ◯ B ◯ C ◯ D ◯

Question Five

A ◯ B ◯ C ◯ D ◯

Question Six

A ◯ B ◯ C ◯ D ◯

Question Seven

A ◯ B ◯ C ◯ D ◯

Question Eight

A ◯ B ◯ C ◯ D ◯

Question Nine

A ◯ B ◯ C ◯ D ◯

Question Ten

A ◯ B ◯ C ◯ D ◯

FORM VI 2 UNIT HALF-YEARLY SOLUTIONS, 2013

Q1. B

Q2. A

Q3. A

Q4. D

Q5. B

Q6. B

Q7. A

Q8. C

Q9. B

Q10. C

$\boxed{10}$

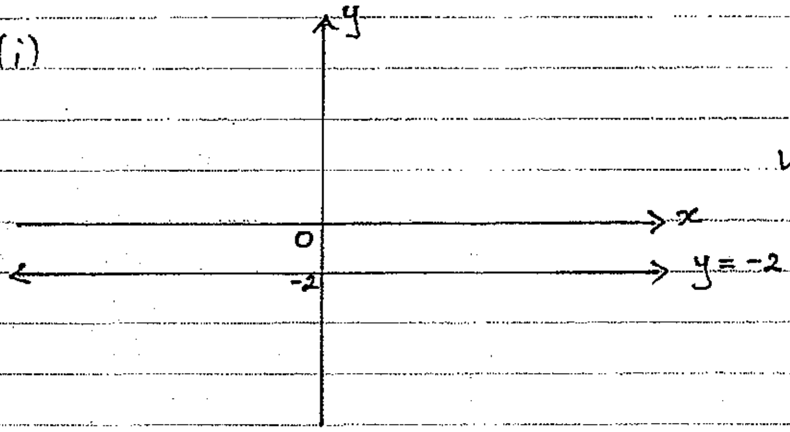
Q11.

(a) $\frac{3e^2}{5} \doteqdot 4.433$ ✓ ①

(b) $\frac{(e^x)^3}{e^{2x}} = \frac{e^{3x}}{e^{2x}}$ ✓

$= e^x$ ✓ ②

(c) (i)



✓

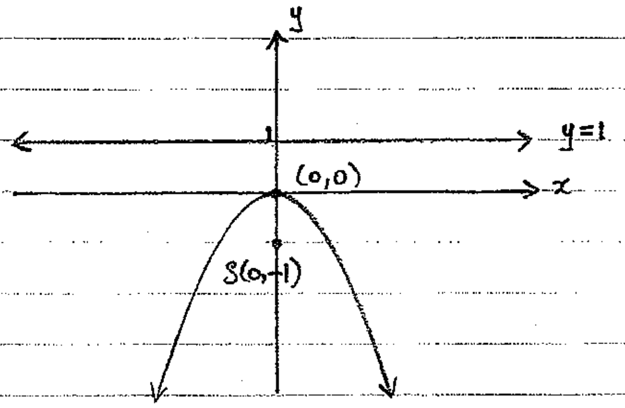
(ii) $y = -2$ ✓ ②

(d) (i) $(0,0)$ ✓

(ii) $4a = 4 \rightarrow a = 1$. Parabola concave down, so

focus at $S(0,-1)$. ✓

(iii) Directrix one unit above vertex: $y = 1$ ✓



Directrix, vertex, focus needed on sketch for mark.

✓

④

(e) (i) $\frac{d}{dx}\left(5x^3-3x^2+9\right) = 15x^2-6x$ ✓

(ii) $\frac{d}{dx} 4e^{5x} = 20e^{5x}$ ✓

(iii) $\frac{d}{dx}(2-3x)^4 = 4(2-3x)^3(-3)$ ✓

$= -12(2-3x)^3$ ✓ (4)

(f) (i) x^3-2x+C (any C, constant) ✓

(ii) $\int x^{-1/2} = 2x^{1/2}+c$ (any c, constant) ✓ (2)

$\boxed{15}$

Q12

(a) $\left(e^x+e^{-x}\right)^2 = e^{2x}+2e^xe^{-x}+e^{-2x}$ ✓

$= e^{2x}+e^{-2x}+2$ ✓ (2)

(b) (i) $\int_{-2}^{2}\left(3x^2+2x\right)dx = x^3+x^2\Big|_{-2}^{2}$ ✓

$= \left(2^3+2^2\right)-\left((-2)^3+(-2)^2\right)$

$= 16$ ✓

(ii) $\int_1^3 \frac{2}{x^2}\,dx = \int_1^3 2x^{-2}\,dx$

$= \left[-2x^{-1}\right]_1^3$ ✓

$= -2\left(\frac{1}{3} - 1\right)$

$= \frac{4}{3}$ or $1\frac{1}{3}$ ✓ (4)

(c) $f'(x) = e^x + 2x \rightarrow f(x) = e^x + x^2 + C$ ✓

Now, $f(0) = e^0 + 0 + C$

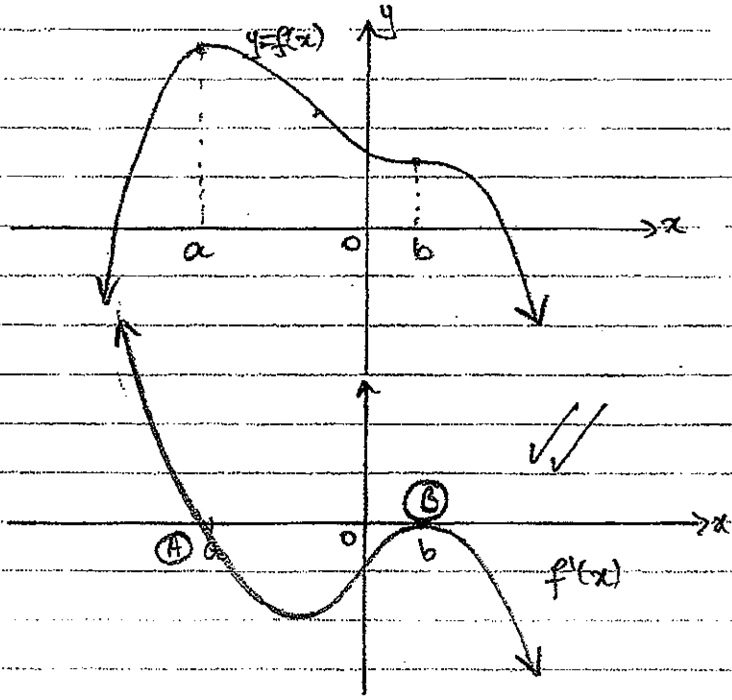
$= 0$

i.e. $1 + C = 0$

$\therefore \quad C = -1$

Hence $f(x) = e^x + x^2 - 1$ ✓ (2)

(d)



1 For any of (A) or (B)

2 for both (A)+(B)

(2)

(e) $$\int_0^2 (e^x - 1)\,dx = \left[e^x - x\right]_0^2$$

$$= (e^2 - 2) - (e^0 - 0)$$

$$= e^2 - 3$$ ②

(f) $$\int_{-1}^{3} (x^2 + x + 1) - (2x^2 - x - 2)\,dx = \int_{-1}^{3} -x^2 + 2x + 3\,dx$$

$$= \left[\frac{-x^3}{3} + x^2 + 3x\right]_{-1}^{3}$$

$$= \left(\left(-\frac{3^3}{3} + 3^2 + 3(3)\right) - \left(-\frac{(-1)^3}{3} + (-1)^2 + 3(-1)\right)\right)$$

$$= \frac{32}{3}$$ ③

$\boxed{15}$

Q13

(a) $y = 3x^3 - 9x + 3$

(i) $\frac{dy}{dx} = 9x^2 - 9$

$= 9(x^2 - 1)$ either

$\frac{d^2y}{dx^2} = 18x$

(ii) Need all x: $\frac{dy}{dx} = 0$.

Hence $9(x^2 - 1) = 0 \rightarrow x = \pm 1$

Corresponding y-coordinates: $y(1) = 3 - 9 + 3 = -3$

$y(-1) = -3 + 9 + 3 = 9$

Stationary points: $(1, -3)$ and $(-1, 9)$

(iii) Nature: $\frac{d^2y}{dx^2} = 18x$, so for:

$x = 1$, $\frac{d^2y}{dx^2} = 18 > 0 \rightarrow$ concave up, local min

$x = -1$, $\frac{d^2y}{dx^2} = -18 < 0 \rightarrow$ concave down, local max.

Hence $(1, -3)$ local min

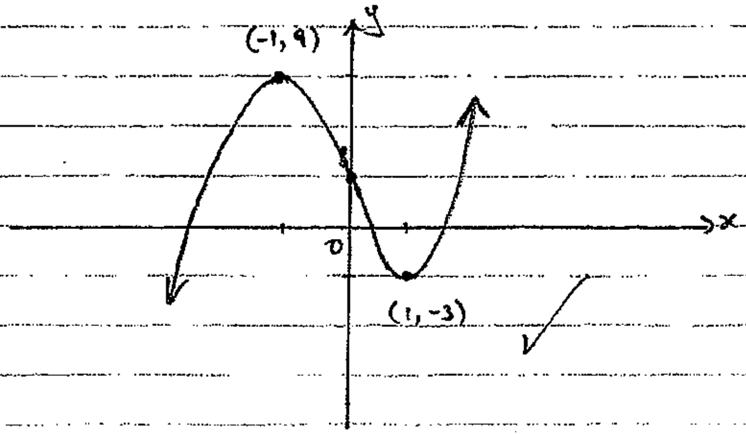
$(-1, 9)$ local max.

OR use table of gradients:

x	-2	-1	0	1	2
$\frac{dy}{dx}$	27	0	-9	0	27

implies $(1, -3)$ local min.
$(-1, 9)$ local max.

(iv)



y-intercept at (0, 3).

Full marks (2) only for both y-intercept & stationary points.

⑧

(b) $\displaystyle\int_{-2}^{2} e^{2x+1}\,dx = \left[\frac{1}{2}e^{2x+1}\right]_{-2}^{2}$

$= \frac{1}{2}\left(e^{5} - e^{-3}\right)$ ②

OR $= \dfrac{e^{8}-1}{2e^{3}}$

(c) $\displaystyle\int_{1}^{5} \frac{1}{x}\,dx \simeq \frac{3-1}{6}\left[1 + 4\times\frac{1}{2} + \frac{1}{3}\right] + \frac{5-3}{6}\left[\frac{1}{3} + 4\times\frac{1}{4} + \frac{1}{5}\right]$

x	1	2	3	4	5
f(x)	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

$= \dfrac{73}{45}$ ②

$\doteqdot 1.622$ (3 d.p.)

(d) $y = e^{3x} \rightarrow y' = 3e^{3x}$

At $x=0$, $y'(0) = 3$

Also, $y(0) = e^{0} = 1$

$\therefore$ line is: $y - 1 = 3(x - 0)$

$y = 3x + 1$ ③

OR $3x - y + 1 = 0$

15

Q14. (a) $C = \frac{2v}{5} + 2000v^{-1}$, $v > 0$.

$$\frac{dC}{dv} = \frac{2}{5} - \frac{2000}{v^2}$$

Extrema for v such that $\frac{dC}{dv} = 0$

Hence,

$$\frac{2}{5} - \frac{2000}{v^2} = 0$$

$$\frac{2000}{v^2} = \frac{2}{5}$$

$$v^2 = 5000$$

$$\therefore v = \pm\sqrt{5000}$$

$$= \pm 50\sqrt{2} \quad \rightarrow \quad v = 50\sqrt{2}\ \text{kmh}^{-1} \ (\because v > 0)$$

OR $v = 70.7\ \text{kmh}^{-1}$ Any

Test nature:

$$\frac{d^2C}{dv^2} = -2000 \times (-2)\, v^{-3}$$

> 0 for all $v > 0$, so global min where $v = 50\sqrt{2}$ km/h.

OR

$\frac{d^2C}{dv^2}$ at $v = 50\sqrt{2}$: $\frac{-2000 \times (-2)}{(50\sqrt{2})^3} > 0$ accepted.

$\therefore$ local minimum where $v = 50\sqrt{2}$ km/h. (3)

Accept approximations too: $50\sqrt{2}$ km/h $\simeq 70.71$ km/h.

OR table of gradients used to test nature of stationary point for one mark.

(b) $y = \sqrt{x}$

Rotation about x-axis: $V = \pi \int_0^4 (\sqrt{x})^2 \, dx$

$= \pi \int_0^4 x \, dx$ ✓

$= \pi \left[\frac{x^2}{2} \right]_0^4$

$= \pi \left[\frac{4^2}{2} - 0 \right]$

$= 8\pi \ u^3$ ✓ 2

(c) $f''(x) = 20(x-1)^2(x-4)$

$f(x)$ will have potential points of inflexion for $f''(x)=0$.

Hence, $20(x-1)^2(x-4)=0$

implies $x=1$ or $x=4$ ✓

Test concavity change:

x	0	1	2	4	5
$f''(x)$	<0	.	<0	.	>0
concavity	⌒	.	⌒	.	⌣

Change in concavity only where $x = 4$.

$\therefore$ $f(x)$ has only one point of inflexion. ✓

2

(d) (i) $\frac{dy}{dx} = e^x(2) + (2x-1)e^x$

$= e^x(2 + 2x - 1)$

$= (2x+1)e^x$

(ii) $\frac{dy}{dx} = \frac{(2x+3)e^x - e^x(2)}{(2x+3)^2}$

$= \frac{e^x(2x+1)}{(2x+3)^2}$

4

(e) (i) $y = e^{x^3} \rightarrow \frac{dy}{dx} = 3x^2e^{x^3}$

$\boxed{\text{OR}}$ $y = e^{x^3}$. Let $u = x^3$ Then $y = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= e^u \cdot 3x^2$

$= 3x^2e^{x^3}$

(ii) $\therefore \int_0^1 3x^2e^{x^3}\,dx = e^{x^3}\Big|_0^1$

$= e^1 - e^0$

$= e - 1$

4

$\boxed{15}$

Q15

(a) $f(x) = \frac{1}{(x+4)^2}$, $x \neq -4$.

$= (x+4)^{-2}$

$\therefore f'(x) = -2(x+4)^{-3}$

& $\therefore f''(x) = \frac{6}{(x+4)^4}$ ✓

Now, $(x+4)^4 > 0$ for all real x, $x \neq -4$. Hence

$\frac{6}{(x+4)^4} > 0$ for all real x, $x \neq -4$. ✓

Therefore $f''(x) > 0$ on this domain,

so $f(x)$ is concave up for all x real,

$x \neq -4$. ✓

3

(b) $y = xe^{-2x} \rightarrow y' = x \cdot (-2)e^{-2x} + e^{-2x} \cdot 1$
$= -2xe^{-2x} + e^{-2x}$ ✓

& $y'' = -2x \cdot (-2)e^{-2x} + e^{-2x} \cdot (-2) + (-2)e^{-2x}$

$= 4xe^{-2x} - 4e^{-2x}$ ✓

Now, $y'' + 4y' + 4y = 4xe^{-2x} - 4e^{-2x} + 4(-2xe^{-2x} + e^{-2x}) + 4(xe^{-2x})$

$= \cancel{4xe^{-2x}} \cancel{-4e^{-2x}} \cancel{-8xe^{-2x}} \cancel{+4e^{-2x}} \cancel{+4xe^{-2x}}$

$= 0$ ✓

3

(c)(i) $\dfrac{l}{2\pi(5)} = \dfrac{\theta}{360} \rightarrow l = \dfrac{10\pi\theta}{360}$

Accept: $l = r\theta \times \dfrac{\pi}{180}$

$= 5\theta \times \dfrac{\pi}{180}$

$= \dfrac{\pi\theta}{36}$

i.e. $l = \dfrac{\pi\theta}{36}$ ✓

(ii) Circumference of base of cone is equal to arc length of sector.

$\therefore \; l = 2\pi r$

Since $l = \dfrac{\pi\theta}{36}$, it follows,

$$\frac{\pi\theta}{36} = 2\pi r$$

$$\therefore r = \frac{\theta}{72}$$ ✓

(iii) By Pythagoras' theorem,

$$5^2 = r^2 + h^2$$

hence $h = \sqrt{25 - r^2}$ ✓

$$= \sqrt{25 - \left(\frac{\theta}{72}\right)^2}$$ ✓

(iv) $V = \dfrac{1}{3}\pi r^2 h$ ✓

$\therefore V = \dfrac{1}{3}\pi\left(\dfrac{\theta}{72}\right)^2\sqrt{25 - \left(\dfrac{\theta}{72}\right)^2}$ or equivalent ✓

(v) $$\frac{dV}{d\theta} = \frac{1}{3}\frac{\pi}{72^2}\left[2\theta\sqrt{25-\theta^2/72^2} + \theta^2 \cdot \frac{1}{2\sqrt{25-\theta^2/75}} \cdot \frac{-2\theta}{72^2}\right]$$ ✓

Set $\frac{dV}{d\theta} = 0$ & solve for θ: implies,

$$2\theta\sqrt{25-\theta^2/72^2} - \frac{\theta^3}{72^2}\frac{1}{\sqrt{25-\theta^2/72^2}} = 0$$

$$\theta\left(2\sqrt{25-\theta^2/72^2} - \frac{\theta^2}{72^2}\frac{1}{\sqrt{25-\theta^2/72^2}}\right) = 0$$

$\therefore$ $\theta = 0$ (reject
↳ cone volume clearly not maximised for $\theta = 0$).

or

$$2\sqrt{25-\theta^2/72^2} = \frac{\theta^2}{72^2\sqrt{25-\theta^2/72^2}}$$

i.e.

$$2\cdot 72^2\left(25-\theta^2/72^2\right) = \theta^2$$

$$2\cdot 25\cdot 72^2 - 2\theta^2 = \theta^2$$

$$\theta^2 = 86400$$

so

$$\theta = 120\sqrt{6} \quad \text{(+ve sol}^{\text{n}}\text{ only)}$$

$$= 293.938\ldots$$

$$\simeq 294^\circ$$ ✓

Test nature of stationary point:

θ	290°	$120\sqrt{6}^\circ$	300°
$\frac{dV}{d\theta}$	≈ 0.026	0	≈ -0.046
slope	/	—	\

$\therefore$ Volume is at a maximum when $\theta^\circ = 120\sqrt{6}^\circ$

or $\theta^\circ \simeq 294^\circ$.

Note: Incorrect response to part (iv) but correct differentiation received full marks.

9

$\boxed{15}$